DYNAMIC EVALUATION OF OVERTURNING MOMENT REDUCTION FACTOR USED IN STATIC SEISMIC LOADING

by

A. Heidebrecht

Department of Civil Engineering and Engineering Mechanics
McMaster University
Hamilton, Ontario

INTRODUCTION

The static seismic loading provisions of the 1975 National Building Code of Canada include, as do most other seismic codes, an overturning moment reduction factor which is multiplied by the statically computed overturning moment to obtain the estimated maximum moment. This factor is an attempt to recognize that the static load distribution which is used to compute the total base shear is likely to overestimate the overturning moment. The reasons for this overestimation include the manner in which the various modes are combined during the dynamic response of the structure and also that the response spectra ordinates differ for the various modal periods. Earlier codes (Ferahian 1970) prescribed relatively low reduction factors, but recent earthquake damage (Hanson and Degenkolb 1969) has indicated that the true overturning moments were not as low as had been estimated. Since that time, most codes have increased the required reduction factor, and several have eliminated the reduction factor entirely, e.g. the 1971 Code of the Structural Engineers Association of California and the 1973 New Zealand Code.

The reduction factors used in the various codes have been based on empirical data and on relatively simple evaluations of the contributing phenomena. Previous investigations into overturning moments (Bustamante 1965; Fenves and Newmark 1969) have emphasized the evaluation of specific buildings and have not included a general evaluation of reduction factors. The purpose of this paper is to evaluate the dynamic shear-moment relationship for uniform planar structures comprising shear walls and frames, and from this compute a dynamically-based moment reduction factor.

DESCRIPTION OF MATHEMATICAL MODEL

The mathematical model used for this evaluation is that of the continuous shear-flexure beam, acting as a vertical cantilever. In this model, the frames are equated to a shear beam with stiffness parameter GA, and the shear walls are equated to a flexural beam with stiffness parameter EI. The basic static and dynamic analysis of this model and the application of the model to practical building structures have been previously published (Heidebrecht and Stafford Smith 1973). The equation of motion for such a structure subjected to earthquake ground motion is given by

$$[1] \quad \frac{\partial^4 x}{\partial z^4} - \alpha^2 \frac{\partial^2 x}{\partial z^2} + \frac{1}{c^2}\frac{\partial^2 x}{\partial t^2} = -\frac{1}{c^2}\ddot{U}_o$$

where x is the lateral motion of the building, z is the height above the base, α^2 = GA/EI, c^2 = EI/ρA, where ρA is the mass of the building per unit height, and $\ddot{U}_o$ is the ground acceleration due to the earthquake.

The solution to Eq. [1] can be expressed in the form

$$[2] \quad x(z,t) = \sum_{i=1}^{\infty} \Psi_i(z) q_i(t)$$

in which $\Psi_i(z)$ is the "i"th mode shape, determined as described by Heidebrecht and Stafford Smith (1973), and $q_i(t)$ is generalized timedependent response function for mode "i". The differential equation governing this response function is given by

$$[3] \quad \ddot{q}_i(t) + \omega_i^2 q_i(t) = -p_i \ddot{U}_o$$

in which ω_i is the "i"th natural frequency, determined as described by Heidebrecht and Stafford Smith (1973), and p_i is the modal participation factor, defined by

$$[4] \quad p_i = \frac{\int_0^H \Psi_i(z)dz}{\int_0^H \Psi_i^2(z)dz}$$

It is useful if the mode shape is normalized so as to make the denominator of Eq. [4] equal to unity; the remainder of this discussion incorporates this normalization.

RESPONSE ANALYSIS

The purpose of this analysis is to determine the maximum values of specific response parameters. In order to do so, the maximum response in each mode will be determined and the resulting modal maxima will then be combined using the root sum square summation (the maximum response in each mode is squared; these squares are summed for all contributing modes and the square root of this sum is the estimated overall maximum response) to obtain an estimate of the overall maximum response. Using the response spectrum approach, the maximum displacements and accelerations in mode "i" are given by

$$[5] \quad x_i^{max}(z) = p_i S_{xi} \Psi_i(z)$$

$$[6] \quad \ddot{x}_i^{max}(z) = p_i S_{ai} \Psi_i(z)$$

in which S_{xi} and S_{ai} are, respectively, the spectral displacement and acceleration ordinates at frequency ω_i. The equivalent inertial loading in mode "i" is given by

$$[7] \quad w_i(z) = \rho A p_i S_{ai} \Psi_i(z)$$

and the resulting maximum base shear and moment in mode "i" can be expressed as

$$[8] \quad V_i = \rho A p_i S_{ai} \int_0^H \Psi_i(z)dz$$

$$[9] \quad M_i = \rho A p_i S_{ai} \int_0^H z\Psi_i(z)dz$$

Including N modes in the total response of the structure, the maximum base shear and moment are then calculated by

$$[10] \quad V_N^* = \sqrt{\sum_{i=1}^{N} V_i^2}$$

$$[11] \quad M_N^* = \sqrt{\sum_{i=1}^{N} M_i^2}$$

REDUCTION FACTOR ANALYSIS

The general definition of the overturning moment reduction factor is the ratio of dynamic base moment to static base moment when the dynamic base shear and the static base shear are taken to be equal. This definition can be expressed in the form

$$[12] \quad J_N = \frac{M_N^*}{V_N^* H} \bigg/ \frac{M_o}{V_o H}$$

in which J_N is the moment reduction factor computed for N modes in the dynamic response, M_o and V_o are, respectively, the base shear and moment computed from the static loading, and H is the height of the structure. Expressing the static non-dimensional moment-shear ratio by

[13] $$a_o = \frac{M_o}{V_o H}$$

allows the reduction factor to be put into the following form

[14] $$J_N = \frac{1}{a_o} \frac{M_N^*}{V_N^* H}$$

For the normal triangular static loading prescribed in the 1975 National Building Code of Canada, as shown in Fig. 1a, $a_o = 2/3$. For slender buildings, the Code requires the addition of a concentrated force at the top of the building, as shown in Fig. 1b. For the maximum value of this top force, equal to fifteen percent of the base shear, $a_o = .715$.

In order to assess the effect of the response spectrum on the moment reduction factor, it is useful to define a constant spectrum reduction factor $\bar{J}_N$, determined by letting S_{ai} = constant for all modes in Eqs. [8] and [9]. In order to determine the effect of including higher modes, the first mode reduction factor is defined by

[15] $$\bar{J}_1 = \frac{1}{a_o} \frac{M_1}{V_1 H}$$

Since M_1 and V_1 in the above equation are determined directly from Eqs. [8] and [9], S_{ai} does not enter into the computation and $\bar{J}_1$ is effectively an expression of the relationship between the static load distribution and the first mode dynamic behaviour.

RESULTS

Calculations of reduction factors have been made for shear-flexure beams with a variety of values of the stiffness ratio α^2. The significant parameter in describing the shear-flexure beam is the non-dimensional parameter αH. Figure 2 shows the mode shapes for different values of αH. When $\alpha H = 0$, the structure is a pure shear-wall. When αH is very large, the structure is a pure frame; for practical computational purposes, the "pure frame" value of αH is taken to be 30. For a mixed frame-shear wall building, the value of $\alpha H = 5$ has been used since that mode shape is very nearly an average of the pure frame and shear wall mode shapes.

Calculations have been made for two different sets of response spectra, in order to assess the effect of different spectra on the moment reduction factor. One of these sets is the set of average response spectra given by Housner (1959), as given in Fig. 3a. The other set of spectra, given in Fig. 3b, are these recommended in the National Building Code of Canada (1975).

Figures 4 and 5 show the moment reduction factors computed for five modes and for two different values of damping for $a_0 = 2/3$; for other values of a_0, the ordinates of these diagrams should be multiplied by $3a_0/2$. The reduction factor specified in the 1975 National Building Code of Canada is also shown in each diagram. It can be seen from these figures that the damping, type of structure, and the response spectrum each have significant effects on the reduction factor. Damping is the least significant of these, although it can contribute to sizeable differences in value in the intermediate period range ($1.0<T<2.5$). The differences between the results for the two sets of response spectra are primarily due to different acceleration bounds. In particular, the 1975 N.B.C. of Canada spectrum does not become asymptotic to the ground acceleration at low periods, whereas the Housner spectrum does.

Fig. 6a shows the variation of the constant spectrum reduction factor $\bar{J}_N$ (N=5) and the first mode reduction factor $\bar{J}_1$ as a function of the parameter αH. The values of $\bar{J}_N$ are identical to the very short period values of J_N, due to the fact that the spectral accelerations are constant in the low period region. From these diagrams, it can be seen that the higher modes alone contribute very little to the reduction factor. The major contribution is therefore due to the changing ordinates of the response spectra associated with the higher modes. The differences between structural types are due primarily to the differing ratios of higher periods to the fundamental periods, and the consequent differences in spectral accelerations at the higher mode periods. Fig. 6b shows the changing ratio of second period to fundamental period for different values of αH.

CONCLUSIONS AND RECOMMENDATIONS

An examination of Figs. 4 and 5 shows that the reduction factors prescribed in the 1975 National Building Code of Canada are conservative for shear wall buildings but may be as much as fifteen percent too low for frame structures having intermediate to long periods. From these results, it would appear that increasing the minimum plateau of the Code reduction factor from 0.8 to 0.9 would provide a safe level of reduction factor for most structures. It should also be possible to allow for reduced factors for shear wall structures provided that the fundamental mode shape is close to that given for

$\alpha H = 0$ in Fig. 2. Further investigations for non-uniform structures are required before more definitive recommendations can be made. It is also true that inelastic response and varying forms of energy absorption within structures may also have a significant effect on the overturning moments which occur during the dynamic response of a structure.

REFERENCES

Bustamante, J.I. 1965. "Seismic Shears and Overturning Moments in Buildings", Proceedings of the Third World Conference on Earthquake Engineering, New Zealand, Vol. III, pp. IV 144 to 160.

Fenves, S.J., and Newmark, N.M. 1969. "Seismic Forces and Overturning Moments in Buildings, Towers and Chimneys", Proceedings of the Fourth World Conference on Earthquake Engineering, Chile, Vol. III, pp. B-5, 1 to 12.

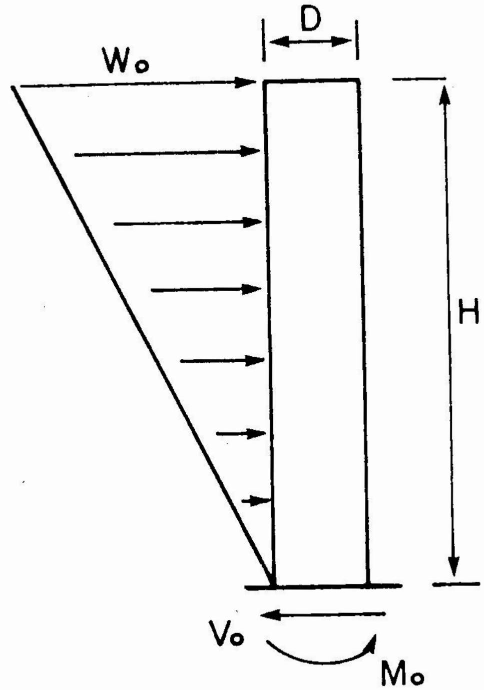
Ferahian, R.H. 1970. "Earthquake Loads", Commentary No. 3, Supplement No. 4, National Building Code of Canada 1970, pp. 579-595.

Hanson, R.D., and Degenkolb, H.J. 1969. "The Venezuela Earthquake July 29, 1967", American Iron and Steel Institute, p. 125.

Heidebrecht, A. C. and Stafford Smith, B. 1973. "Approximate Analysis of Tall Wall-Frame Structures", Journal of the Structural Division, A.S.C.E., Vol. 99, No. ST 2, pp. 199-221.

Housner, G.W. 1959. "Behavior of Structures During Earthquakes", Journal of the Engineering Mechanics Division, A.S.C.E., Vol. 8J, No. EM4, pp. 109-129.

National Building Code of Canada 1975. "Commentary K: Dynamic Analysis for the Seismic Response of Buildings".

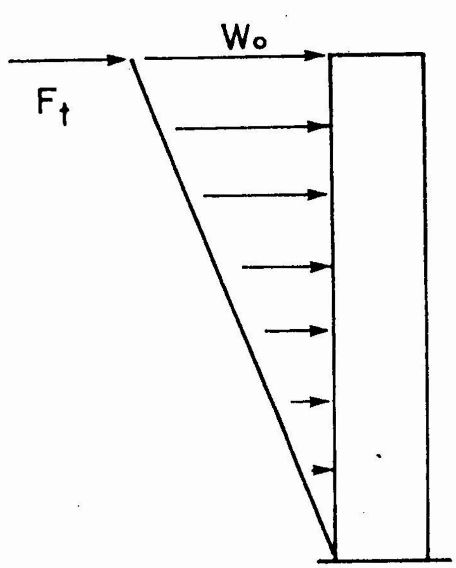


$$V_o = \frac{W_o H}{2}$$

$$M_o = \frac{W_o H^2}{3}$$

$$a_o = \frac{M_o}{V_o H} = \frac{2}{3}$$

a) Normal Structures (H/D ≤ 3)



$$V_o = \frac{W_o H}{2} + F_t$$

$$M_o = \frac{W_o H^2}{3} + F_t H$$

For max value $F_t = .15\,V_o$,

$a_o = .715$

b) Slender Structures (H/D > 3)

FIGURE 1 STATIC SEISMIC LOADING,
NATIONAL BUILDING CODE OF CANADA

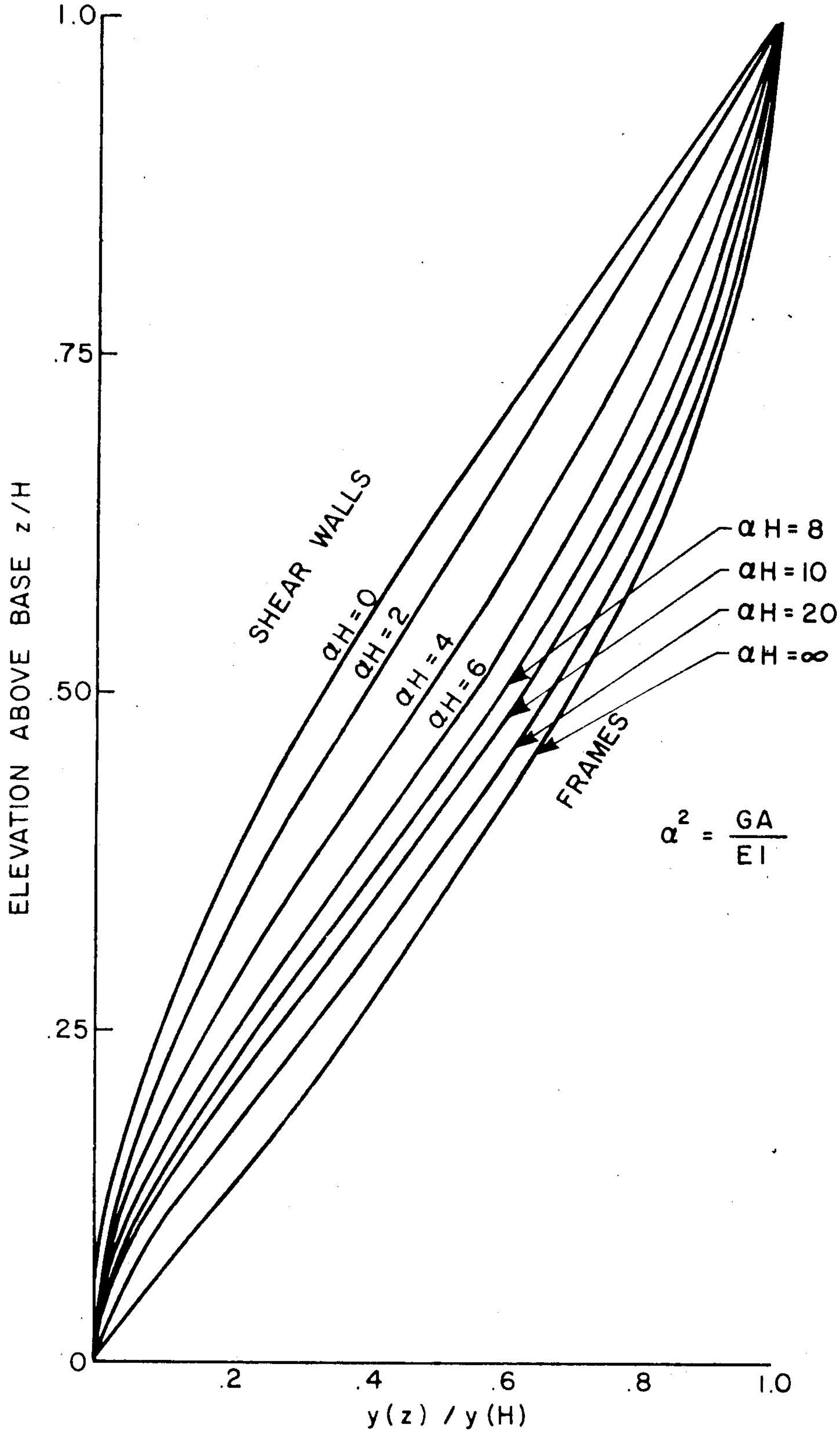
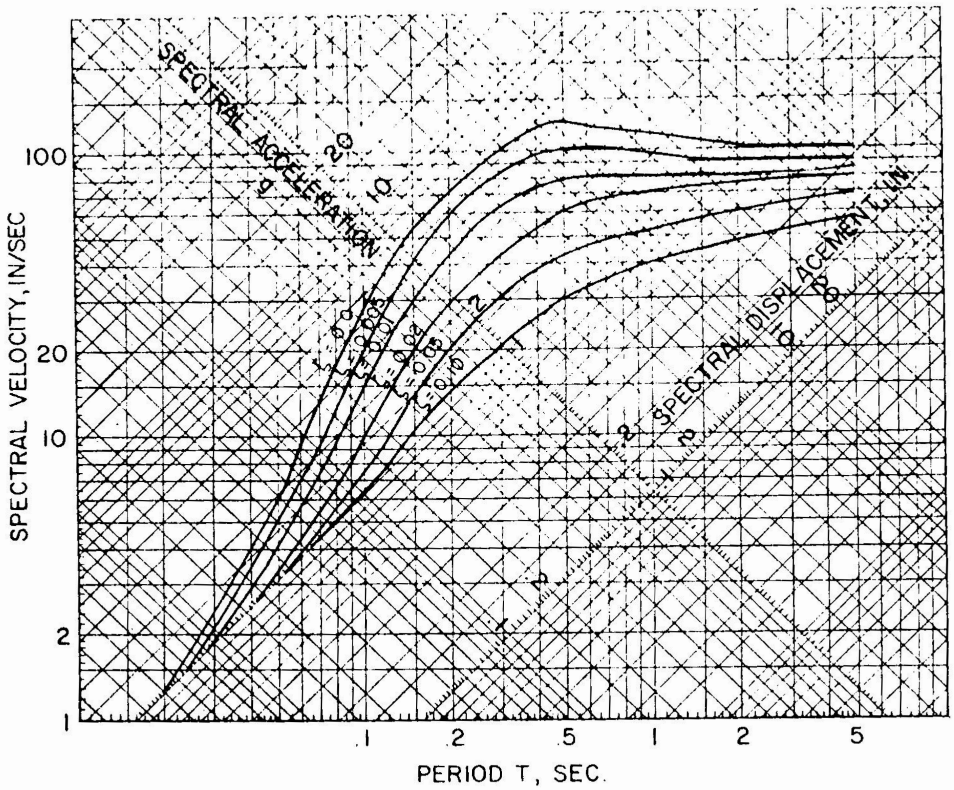
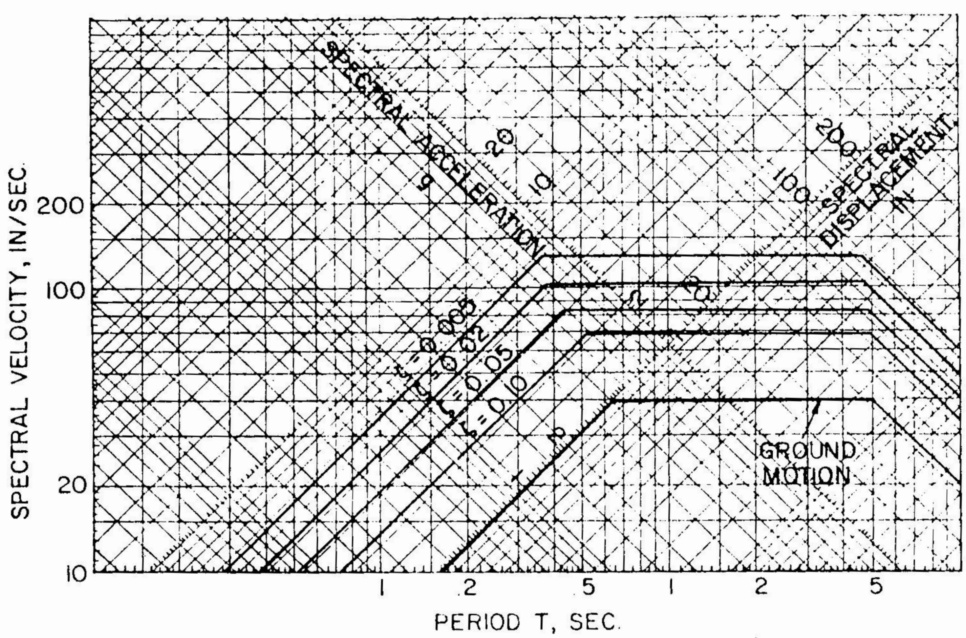


FIGURE 2 FUNDAMENTAL MODE SHAPES



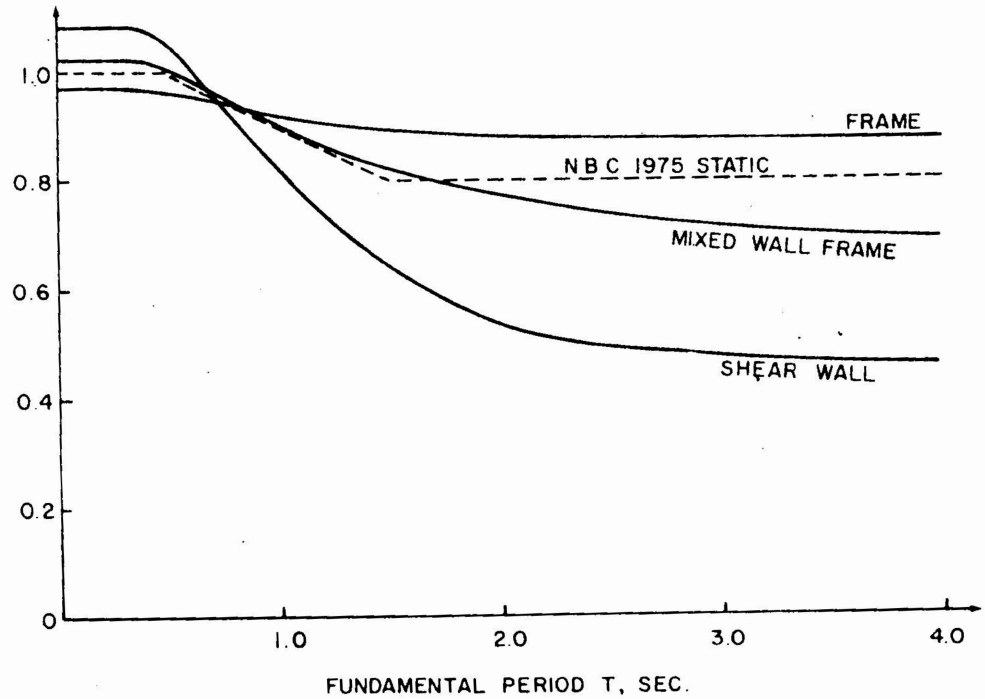
a) Housner's Average Response Spectra, for 1g Max Ground Acceleration



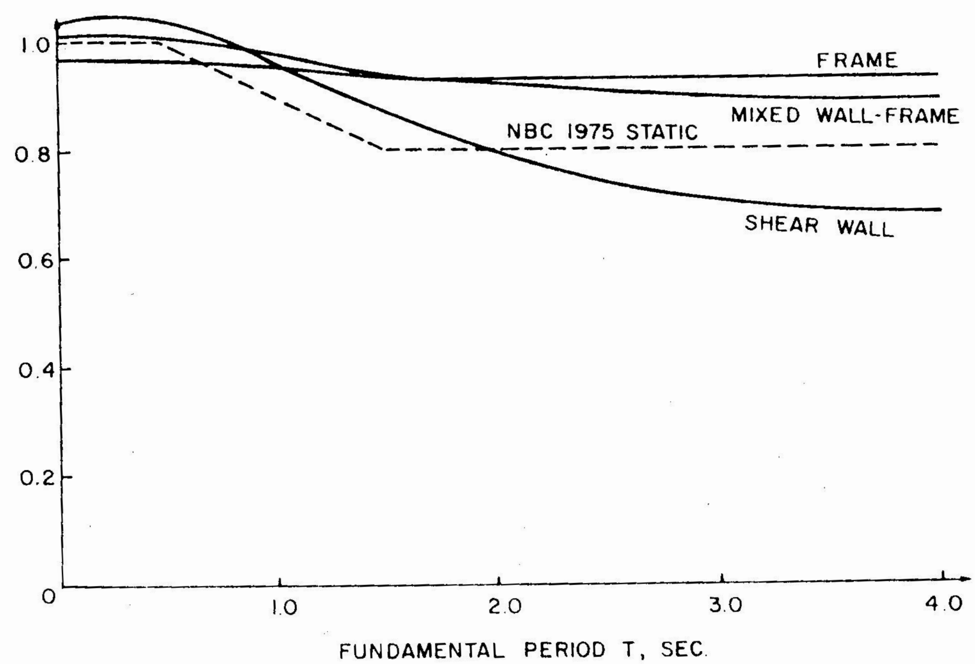
b) Responce Spectra From 1975 National Building Code of Canada, for 1g Max Ground Acceleration

(ζ = Decimal Percentage of Critical Damping)

FIGURE 3 DESIGN RESPONSE SPECTRA

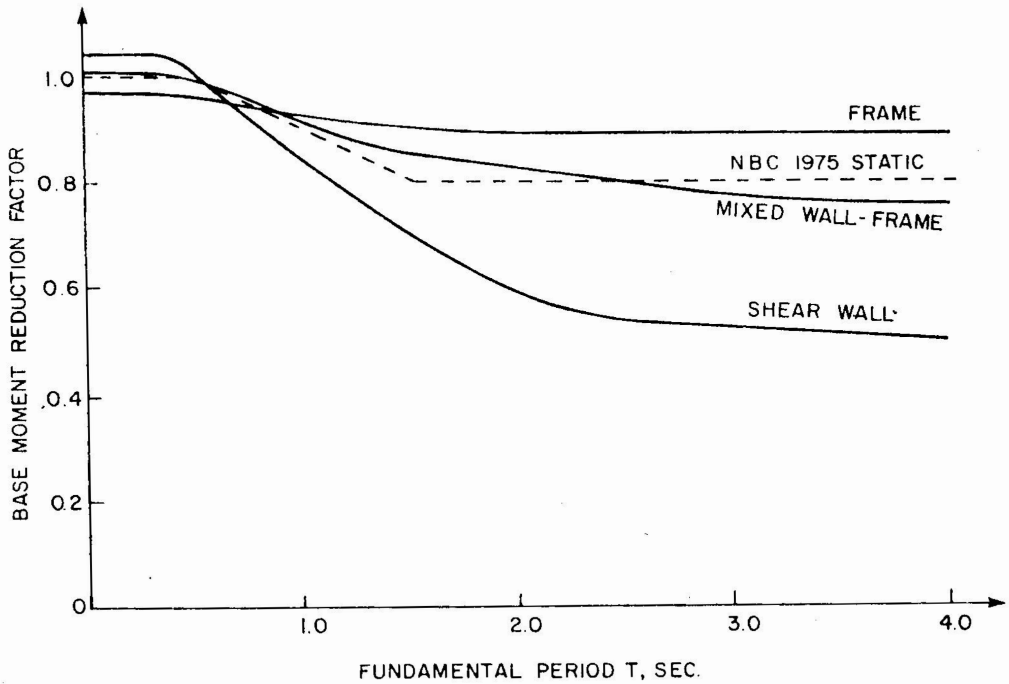


a) Zero Damping

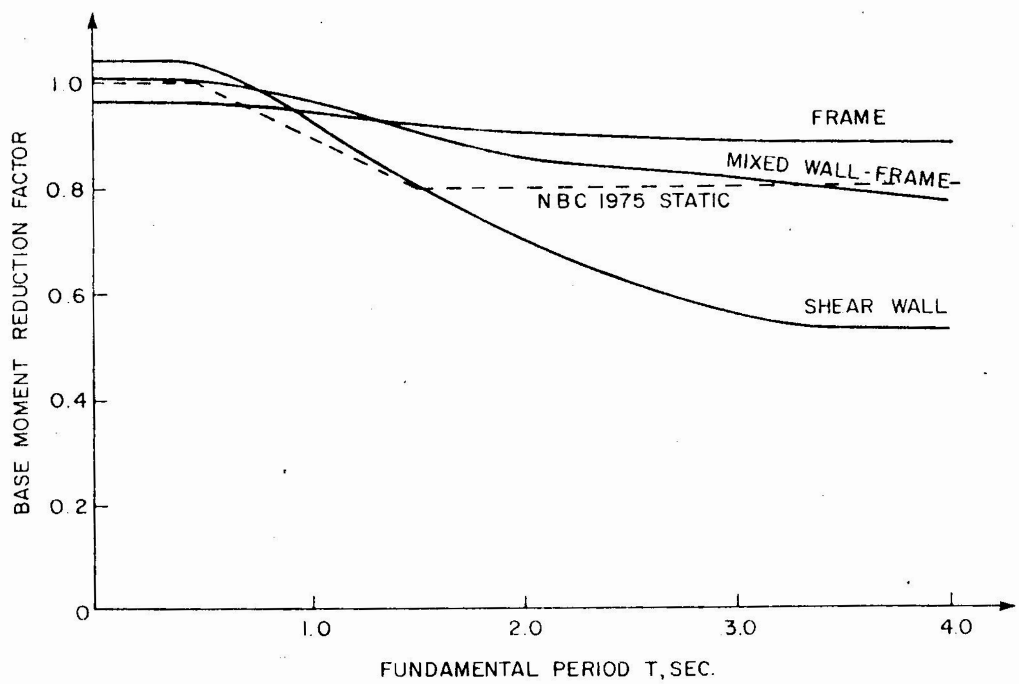


b) Ten Percent Damping

FIGURE 4 MOMENT REDUCTION FACTORS FOR FIVE MODES, USING HOUSNER'S AVERAGE SPECTRA



a) Zero Damping



b) Ten Percent Damping

FIGURE 5 MOMENT REDUCTION FACTORS FOR FIVE MODES, USING SPECTRA RECOMMENDED IN THE 1975 NATIONAL BUILDING CODE OF CANADA

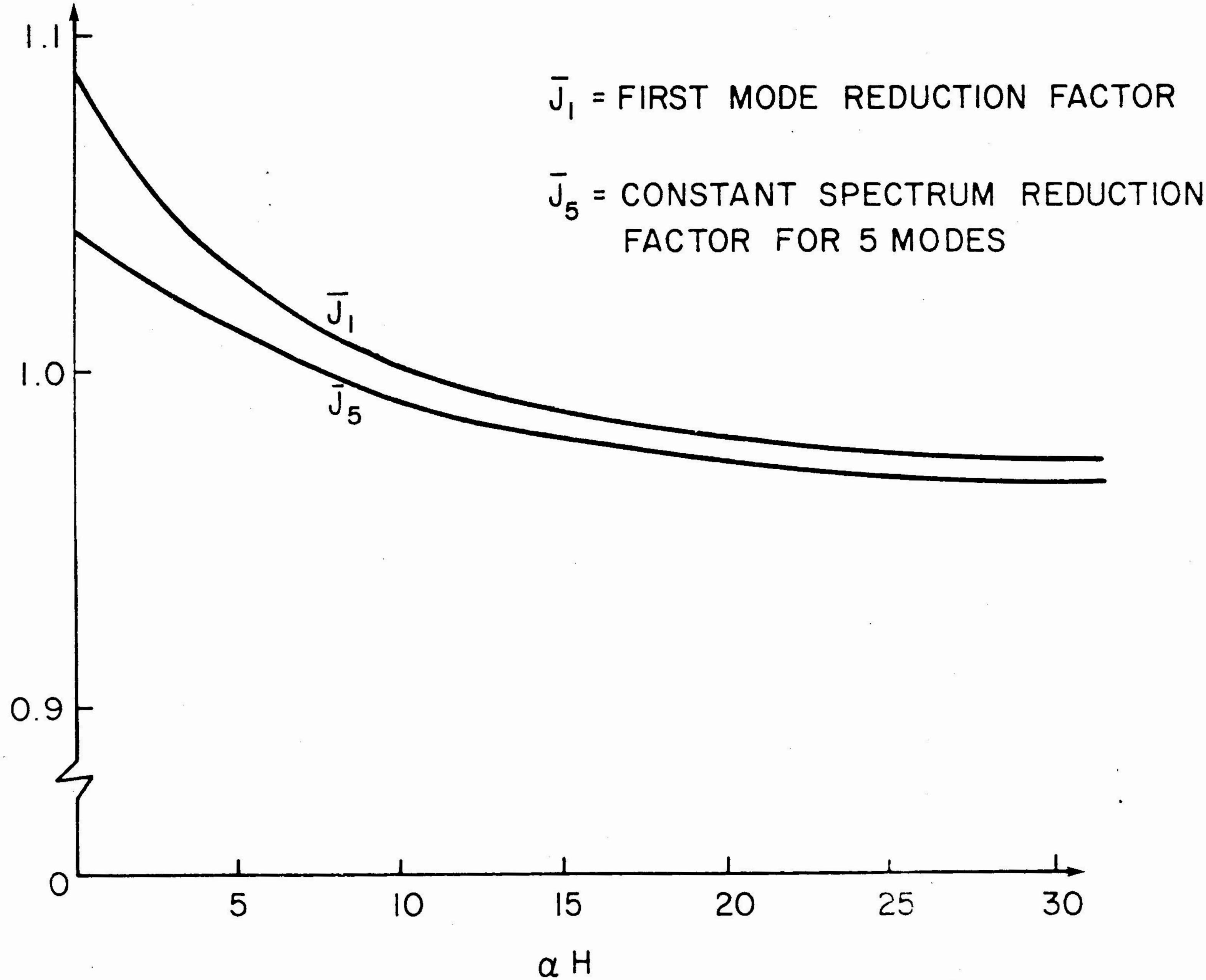


FIGURE 6 CONSTANT SPECTRUM REDUCTION FACTORS

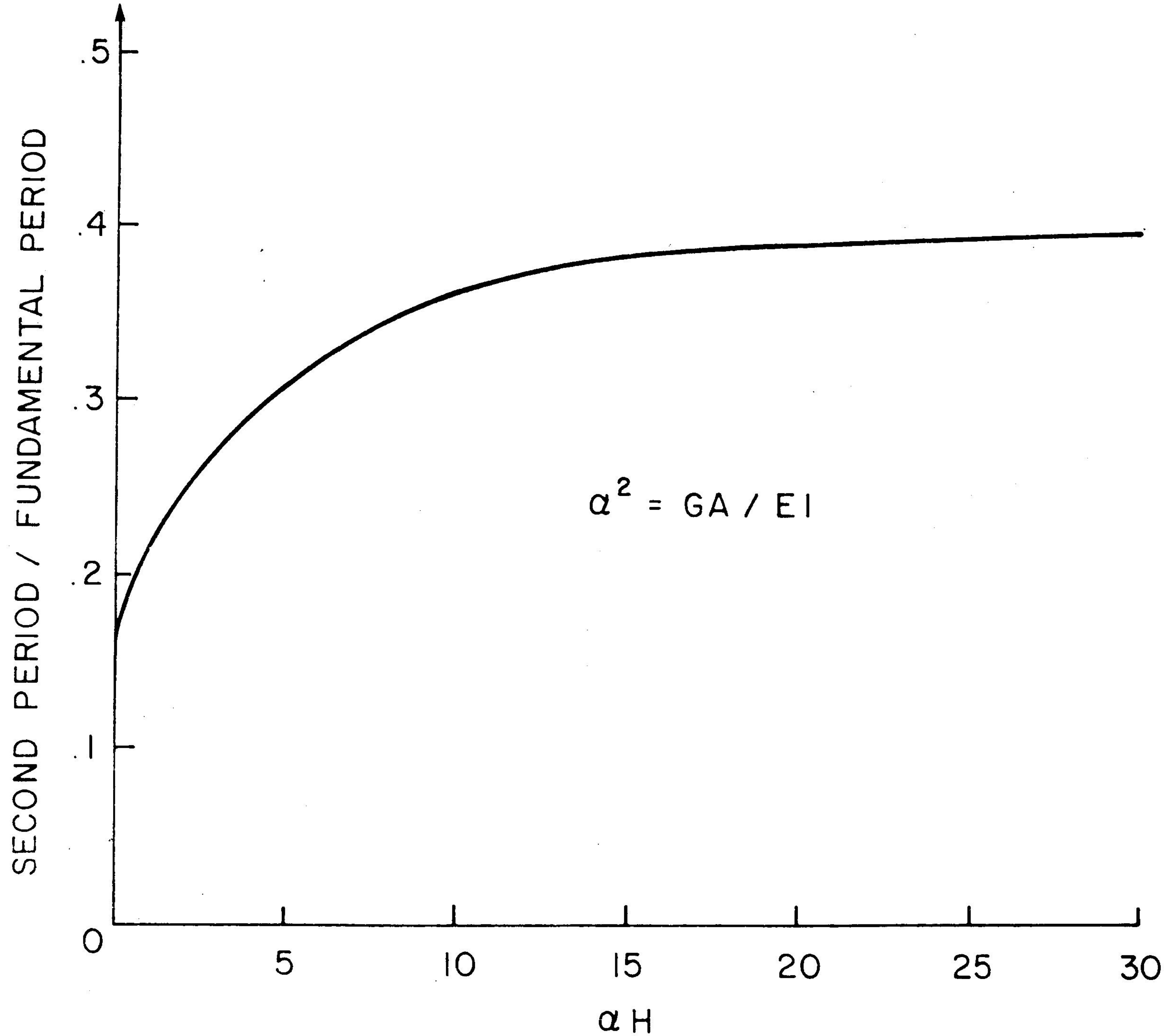


FIGURE 7 RATIO OF SECOND TO FUNDAMENTAL NATURAL PERIOD